

## Polynomial long division – Stages 1 to 10

$$x + 2) \overline{x^3 + x^2 - 1}$$

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$$x + 2 \overline{) x^3 + x^2 - 1}$$

$x^2$

## Polynomial long division – Stages 1 to 10

$$\begin{array}{r} x^2 \\ x + 2 \overline{) x^3 + x^2 - 1} \\ \underline{-x^3 - 2x^2} \phantom{- 1} \\ \phantom{x^3 +} 3x^2 - 1 \phantom{- 1} \end{array}$$

## Polynomial long division – Stages 1 to 10

$$\begin{array}{r} x^2 \\ x + 2 \overline{) x^3 + x^2 - 1} \\ \underline{- x^3 - 2x^2} \phantom{- 1} \\ - x^2 \phantom{- 1} \end{array}$$

## Polynomial long division – Stages 1 to 10

$$\begin{array}{r} x^2 - x \\ x + 2 \overline{) x^3 + x^2 - 1} \\ \underline{- x^3 - 2x^2} \phantom{- 1} \\ - x^2 \phantom{- 1} \end{array}$$

## Polynomial long division – Stages 1 to 10

$$\begin{array}{r} x^2 - x \\ x + 2 \overline{) x^3 + x^2 - 1} \\ \underline{- x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \end{array}$$

## Polynomial long division – Stages 1 to 10

$$\begin{array}{r} x^2 - x \\ x + 2 \overline{) x^3 + x^2 - 1} \\ \underline{- x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \end{array}$$

## Polynomial long division – Stages 1 to 10

$$\begin{array}{r} x^2 - x + 2 \\ x + 2 \overline{) x^3 + x^2 - 1} \\ \underline{- x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \end{array}$$



## Polynomial long division – Stages 1 to 10

$$\begin{array}{r} x^2 - x + 2 \\ x + 2 \overline{) x^3 + x^2 - 1} \\ \underline{-x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \\ \underline{-2x - 4} \\ \phantom{2x} -5 \end{array}$$



## Polynomial long division – Stages 1 to 11

style=B

$$x^3 + x^2 - 1 = (x + 2) ( \quad )$$

## Polynomial long division – Stages 1 to 11

style=B

$$x^3 + x^2 - 1 = (x + 2)(x^2 \quad \quad \quad )$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline \end{array} \quad - 1 = (x + 2)(x^2 \quad )$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \end{array} - 1 = (x + 2)(x^2 \quad )$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \end{array} \quad - 1 = (x + 2)(x^2 - x \quad )$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \\ \phantom{-x^2} x^2 + 2x \\ \hline \end{array} - 1 = (x + 2)(x^2 - x \quad )$$



## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 - 1 \\ - x^3 - 2x^2 \\ \hline -x^2 \\ \phantom{-x^2} + 2x \\ \hline 2x - 1 \end{array} = (x + 2)(x^2 - x - 1)$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \\ \phantom{-x^2} + 2x \\ \hline 2x - 1 \end{array} - 1 = (x + 2)(x^2 - x + 2)$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 \quad - 1 = (x + 2)(x^2 - x + 2) \\ - x^3 - 2x^2 \\ \hline \quad - x^2 \\ \quad \quad x^2 + 2x \\ \quad \quad \quad \hline \quad \quad \quad 2x - 1 \\ \quad \quad \quad - 2x - 4 \\ \quad \quad \quad \quad \hline \end{array}$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 - 1 \\ - x^3 - 2x^2 \\ \hline -x^2 - 1 \\ \phantom{-x^2 - 1} + x^2 + 2x \\ \hline \phantom{-x^2 - 1} 2x - 1 \\ \phantom{-x^2 - 1} - 2x - 4 \\ \hline \phantom{-x^2 - 1} - 5 \end{array}$$

## Polynomial long division – Stages 1 to 11

style=B

$$\begin{array}{r} x^3 + x^2 - 1 \\ - x^3 - 2x^2 \\ \hline -x^2 - 1 \\ \phantom{-x^2 - 1} + x^2 + 2x \\ \hline \phantom{-x^2 - 1} 2x - 1 \\ \phantom{-x^2 - 1} - 2x - 4 \\ \hline \phantom{-x^2 - 1} - 5 \end{array}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\left( x^3 + x^2 - 1 \right) \div (x + 2) = \quad + \frac{\quad}{x + 2}$$

## Polynomial long division – Stages 1 to 11

style=C

$$(x^3 + x^2 - 1) \div (x + 2) = x^2 + \frac{\quad}{x + 2}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\left( \begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline \end{array} - 1 \right) \div (x + 2) = x^2 + \frac{\quad}{x + 2}$$



## Polynomial long division – Stages 1 to 11

style=C

$$\left( \begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \end{array} - 1 \right) \div (x + 2) = x^2 + \frac{\quad}{x + 2}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\left( \begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \end{array} - 1 \right) \div (x + 2) = x^2 - x + \frac{\quad}{x + 2}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\left( \begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \\ \phantom{-x^2} + 2x \\ \hline \end{array} - 1 \right) \div (x + 2) = x^2 - x + \frac{\phantom{-x^2} + 2x}{x + 2}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\begin{array}{r} (x^3 + x^2 - 1) \div (x + 2) = x^2 - x + \frac{\quad}{x + 2} \\ \underline{-x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \end{array}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\begin{array}{r} (x^3 + x^2 - 1) \div (x + 2) = x^2 - x + 2 + \frac{\quad}{x + 2} \\ \underline{-x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \end{array}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\begin{array}{r} (x^3 + x^2 - 1) \div (x + 2) = x^2 - x + 2 + \frac{\quad}{x + 2} \\ \underline{-x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \\ \underline{-2x - 4} \phantom{- 1} \end{array}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\begin{array}{r} (x^3 + x^2 - 1) \div (x + 2) = x^2 - x + 2 + \frac{\quad}{x + 2} \\ \underline{-x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \\ \underline{-2x - 4} \\ -5 \end{array}$$

## Polynomial long division – Stages 1 to 11

style=C

$$\begin{array}{r} (x^3 + x^2 - 1) \div (x + 2) = x^2 - x + 2 + \frac{-5}{x + 2} \\ \underline{-x^3 - 2x^2} \phantom{- 1} \\ -x^2 \phantom{- 1} \\ \underline{x^2 + 2x} \phantom{- 1} \\ 2x - 1 \\ \underline{-2x - 4} \\ -5 \end{array}$$



## Polynomial long division – Stages 1 to 11

style=D

$$x^3 + x^2 \quad - 1 \left| x + 2 \right. \underline{\hspace{2cm}}$$

## Polynomial long division – Stages 1 to 11

style=D

$$x^3 + x^2 \quad - 1 \left| \begin{array}{l} x + 2 \\ \hline x^2 \end{array} \right.$$

## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline \end{array} \qquad - 1 \left| \begin{array}{l} x + 2 \\ \hline x^2 \end{array} \right.$$

## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \end{array} \qquad -1 \left| \begin{array}{l} x + 2 \\ \hline x^2 \end{array} \right.$$

## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \end{array} \qquad -1 \left| \begin{array}{r} x + 2 \\ \hline x^2 - x \end{array} \right.$$

## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \\ - x^3 - 2x^2 \\ \hline -x^2 \\ \phantom{-x^2} x^2 + 2x \\ \hline \end{array} \quad -1 \left| \begin{array}{r} x + 2 \\ \hline x^2 - x \end{array} \right.$$



## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \quad - 1 \mid x + 2 \\ - x^3 - 2x^2 \quad \mid \hline \hline - x^2 \quad \mid x^2 - x + 2 \\ \quad x^2 + 2x \quad \mid \hline \hline \quad \quad \quad 2x - 1 \end{array}$$



## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \quad - 1 \mid x + 2 \\ - x^3 - 2x^2 \quad \mid \hline \hline - x^2 \quad \mid \\ \quad x^2 + 2x \quad \mid \\ \quad \hline \quad \quad 2x - 1 \quad \mid \\ \quad \quad - 2x - 4 \quad \mid \\ \quad \quad \hline \end{array}$$

## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \quad - 1 \mid x + 2 \\ - x^3 - 2x^2 \quad \mid x^2 - x + 2 \\ \hline - x^2 \quad \mid \\ \quad x^2 + 2x \quad \mid \\ \hline \quad \quad 2x - 1 \quad \mid \\ \quad - 2x - 4 \quad \mid \\ \hline \quad \quad \quad - 5 \end{array}$$

## Polynomial long division – Stages 1 to 11

style=D

$$\begin{array}{r} x^3 + x^2 \quad - 1 \mid x + 2 \\ - x^3 - 2x^2 \quad \mid \hline \hline \quad - x^2 \quad \mid \\ \quad \quad x^2 + 2x \quad \mid \hline \quad \quad \quad 2x - 1 \quad \mid \\ \quad \quad \quad - 2x - 4 \quad \mid \hline \quad \quad \quad \quad - 5 \quad \mid \end{array}$$

## Horner's scheme – Stages 1 to 8

$$-2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ \hline \end{array} \right.$$



## Horner's scheme – Stages 1 to 8

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & & \\ \hline & 1 & & & \end{array}$$

## Horner's scheme – Stages 1 to 8

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & & \\ \hline & 1 & -1 & & \end{array}$$

## Horner's scheme – Stages 1 to 8

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & \\ \hline & 1 & -1 & & \end{array}$$



## Horner's scheme – Stages 1 to 8

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & \\ \hline & 1 & -1 & 2 & \end{array}$$

## Horner's scheme – Stages 1 to 8

$$-2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 \end{array} \right.$$

## Horner's scheme – Stages 1 to 8

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 & -5 \end{array}$$

## Horner's scheme – Stages 1 to 8

tutor=true,resultstyle=\color{blue}

$$-2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ \hline \end{array} \right.$$

## Horner's scheme – Stages 1 to 8

tutor=true,resultstyle=\color{blue}

$$\begin{array}{cccc} & 1 & 1 & 0 & -1 \\ -2 & | & & & \\ & \downarrow & & & \\ & 1 & & & \end{array}$$

## Horner's scheme – Stages 1 to 8

tutor=true,resultstyle=\color{blue}

$$\begin{array}{r|rrrr} & 1 & 1 & 0 & -1 \\ -2 & & -2 & & \\ \hline & 1 & & & \end{array}$$

1  $\cdot (-2)$

## Horner's scheme – Stages 1 to 8

tutor=true,resultstyle=\color{blue}

$$\begin{array}{r} -2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ & -2 & & \\ \hline & 1 & -1 & \end{array} \right. \end{array}$$

## Horner's scheme – Stages 1 to 8

tutor=true,resultstyle=\color{blue}

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & \\ \hline & 1 & -1 & 2 & \end{array}$$

↗ (-2)



## Horner's scheme – Stages 1 to 8

tutor=true,resultstyle=\color{blue}

$$-2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ & -2 & 2 & + \\ \hline & 1 & -1 & 2 \end{array} \right.$$

## Horner's scheme – Stages 1 to 8

tutor=true,resultstyle=\color{blue}

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 & \cdot(-2) \end{array}$$



## Horner's scheme – Rule the result

resultbottomrule,resultleftrule,resulttrightrule

$$-2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 \end{array} \right. \boxed{-5}$$

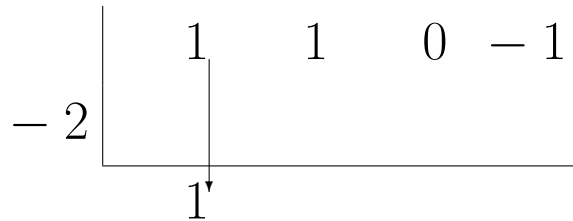
## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3

$$-2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ \hline \end{array} \right.$$

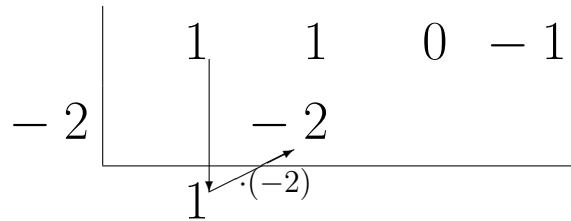
## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3



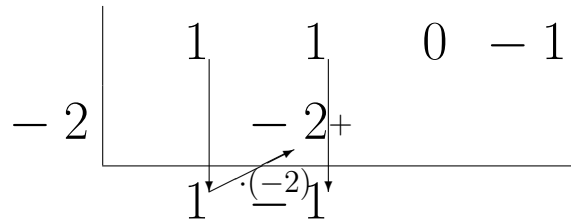
## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3



## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3





## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3

$$\begin{array}{r|rrrr} & 1 & 1 & 0 & -1 \\ -2 & & -2 & +2 & \\ \hline & 1 & -1 & 2 & \end{array}$$

Diagram illustrating the first two stages of Horner's scheme for the polynomial  $x^3 + x^2 - 1$  with root  $-2$ . The diagram shows the coefficients  $1, 1, 0, -1$  and the root  $-2$  on the left. The process involves multiplying the root by the current coefficient and adding the result to the next coefficient. The intermediate results are shown as  $-2$  and  $2$  in the second row. The final result is  $1$  in the third row. The operations are indicated by arrows and the expression  $\cdot(-2)$ .

## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3

$$\begin{array}{r} -2 \left| \begin{array}{cccc} 1 & 1 & 0 & -1 \\ & -2 & 2 & \\ & & & 2 \end{array} \right. \\ \hline \begin{array}{cccc} & 1 & -1 & \end{array} \end{array}$$

(Note: An arrow labeled  $\cdot(-2)$  points from the  $-2$  in the second row to the  $1$  in the third row.)

## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3

$$\begin{array}{r|rrrr} & 1 & 1 & 0 & -1 \\ -2 & & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 & -4 \end{array}$$

*(Note: The original image contains handwritten annotations: arrows pointing from the bottom row to the top row with the label  $\cdot(-2)$ , and a plus sign next to the 2 in the second row.)*

## Horner's scheme – Stages 1 to 8

tutor=true,tutorlimit=3

$$\begin{array}{r|cccc} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 & -5 \end{array}$$

(-2)

## Horner's scheme: Some more options

`showbase=top,showbasesep=false`

$$\begin{array}{rcccc} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 & -5 \end{array}$$